Foreword

Today we live in a paradoxical situation. Mathematics, as a tool for learning rigorous and logical thinking, is irreplaceable; it helps to develop intuition, imagination, and a critical sense. It is also an international language and an important element of our culture. Moreover, by its interactions with other sciences, mathematics is playing an increasing role in designing and developing objects for our daily life. But most people tend not to be aware of this, and for them mathematics has lost its meaning. It is now sometimes fashionable, even among those in responsible positions, to boast at being "zero in math", or to challenge its usefulness.

One can find explanations for this paradox and incomprehension, which are specific to mathematics. It is a discipline which draws on its links with other sciences and the real world, but also enriches itself: its theories are not superseded; they build upon one another. Conversely, even if a large number of researchers in mathematics are interested, above all, in the intellectual and even the aesthetic side of their discipline, sometimes applications emerge in unexpected ways. Thus, applications enrich research, but by themselves cannot direct it.

This subtle balance between internal and external factors of development must be preserved at all costs. Trying to limit mathematical research to its potential applications would be tantamount to its disappearance. On the other hand, stressing, the study of structures and the internal dynamics of the discipline, as was done in French mathematics for several decades beginning in 1940, resulted in delaying the development of applied mathematics in France, in contrast to what happened at the same time in the United States and the Soviet Union. The factors leading to progress are very often at the frontiers of the discipline.

We are delighted that today mathematics has restored, and sometimes created, strong links with other sciences and with many sectors of the economy. The line between pure mathematics and applied mathematics has become blurred: the most fundamental mathematics is being used to solve more and more difficult problems. Thus, fields such as algebraic geometry and the theory of numbers have found unexpected applications in coding theory and in cryptography. In the same way, the strong links between mathematics and finance have helped in the evaluation, and even in the creation, of increasingly complex financial products in response to the needs and demands of the economic players.

However, there still remains much to to be done to change the image of mathematics, and to make the assets and attractions of mathematics and its applications more well-known. The goal of this document is to make mathematics familiar in its various aspects - scientific, technical, cultural, sociological; to underline the diversity and universality of a discipline which has close ties with physics, chemistry, economics and biology, as well as with history, music and painting. Mathematics is everywhere. Without it there would be no computers, no software systems, no mobile phones, no design workshops for car and aeronautical manufacturers, no satellite localisation systems, signal processing, genome decoding, weather forecasting, cryptography, smart cards, or robots.

Beyond its role as an academic science and a basic learning tool in schools, mathematics is omnipresent in today's society. It follows, accompanies, and sometimes precedes the current scientific and technological advances, which rely on the latest results in contemporary fundamental research, as much as they benefit from the accumulated discoveries of the past. Lastly, the need for mathematics grows with the acceleration in technological creation and change. One cannot do without it when confronted with the need to develop, control, or analyse increasingly complex systems.

This need has been understood in the United States, and the NSF (the National Science Foundation, the federal organisation in charge of distributing university research grants) decided in 2000 to substantially increase its financial support for mathematics. Our chance lies in the fact that in France the teaching of mathematics remains one of the best in the world, and that our scientists and engineers are highly regarded internationally. The number of Fields medals, equivalent to the Nobel Prize which does not exist for mathematics, won by Frenchmen is testimony to this. Recently, on the occasion of the third European Congress of Mathematics, which was held in Barcelona in July 2000, five of the ten award winners were French mathematicians. Let us maintain this level of excellence.

Mireille Martin-Deschamps
President of the SMF from 1998 to 2001

Patrick Le Tallec
President of the SMAI from 1999 to 2001